Chapter 7

Finite Element Modeling of Composite Wind Turbine Blades

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I. INTRODUCTION

This chapter involves the finite element modelling of composite wind turbine blades. The two dimensional formulations for plates and shells are based on displacement based methods as opposed to force based methods. A brief background of the finite element procedures is discussed. The governing equations of motion for plate problems are covered. The two dimensional shell analyses are based on flat-facetted, doubly-curved and three dimensional degenerated concepts. Exact three dimensional solutions for plates and shells are included. Finally plane stress, plane strain and generalised plane strain finite element formulations for composite material parts in a composite wind turbine blade are discussed.

II. BACKGROUND TO THE MODELLING CONCEPT

Composite materials possess certain inherent characteristics, which give rise to many challenging issues in wind turbine blades. The advantages of fibre-reinforced polymer (FRP) composites is that the interaction of the polymer matrix with reinforced fibres of high strength and stiffness governs the structural integrity of composites in wind turbine blades and therefore helps in achieving the desired performance. Since mechanical performance and structural integrity of FRP composite material depend on the effectiveness of the bond between resin and fibre in transferring stress across the interface, the selectivity of resin systems for improved fibre-matrix interfacial strength assumes paramount significance. Although the incorporation of fibres dominates the wear and friction processes of FRP composites, the role of the resin matrices is almost equally important. Firstly because matrix characteristics greatly control the interfacial phenomena and secondly because the matrix serves as a medium that transfers the load to the fibres and separates the individual fibres thereby preventing any brittle crack. Hence, studies of variability of materials and defects have been an intense field of research in most practical wind turbine applications. The finite element method due to its versatility is a proven tool to deal with various issues in wind turbine blades.

The finite element method [1-2] simulates the behaviour of a real structure by approximating it with that of a model composed of sub-regions or ‘elements’ in which the displacement field is restricted to a linear combination of pre-selected displacement patterns or ‘shape functions’. The configuration of the model is therefore specified by the magnitudes of the generalised coordinates associated with the shape functions. The configuration which minimizes the potential energy is determined and this configuration is then interpreted as an approximation to the configuration of the structure. The success or failure of the model to represent the conditions in the structure depends primarily on the set of shape functions selected and the compatibility conditions imposed along the boundaries of the elements.

Certain minimum conditions may be set down in an attempt to ensure that the behaviour of the model is a close approximation to the behaviour of the structure. These are summarized as:

1) Completeness requirements [3-7]. The shape function should include rigid body displacements and the constant-strain states associated with the problem under investigation.

2) Continuity requirements [3-7]. The configuration produced by the shape functions should satisfy the minimum internal and external compatibility conditions which are associated with the problem under investigation. Boundary compatibility should be maintained along the total perimeter of the element.

A systematic method for selecting a set of shape functions satisfying the minimum conditions has been successfully introduced based on the utilization of natural coordinates [3-7], interpolating functions [3-7] and complete polynomials [3-7]. The generalised coordinates are identified as the physical displacement quantities at the nodes and the formulation is simplified relating generalised coordinates and nodal displacements.
The finite elements commonly used in structural mechanics are based assumed displacement fields [3-7] and assumed stress fields [3-7]. In the assumed displacement based finite element formulation, the displacements within an element are adequately described by a simple polynomial that satisfies the potential energy principle [3-7]. In the assumed stress finite element formulation [3-7], the stress field must satisfy the equilibrium equation so that it is convenient to begin with a stress function. Hence generalised force degrees of freedom (dof) are primary variables as opposed to displacement degrees of freedom in displacement based FE formulations. In the mixed finite element formulations [3-7], both force and displacement dof are primary unknowns. Hybrid finite element formulations [3-7] maintain an assumed stress field within the element and assumed displacement patterns on their boundaries. In the subsequent sections, finite elements for plates and shells based on the assumed displacement fields in two and three dimensional theories are discussed.

III. TWO DIMENSIONAL FE FORMULATIONS FOR SANDWICH PLATES

Sandwich plates [8-10] in thick and thin regimes develop both membrane, bending and shear stresses under static and dynamic environments. Hence transverse shear deformations [11-17] are to be included in the development of the resulting two dimensional finite element formulations.

The displacement field based on a first order shear deformation theory [7] is expressed (see Fig. 1) as

\[
  u_i = \sum_{i=1}^{4} (u_{oi} N_i + z \psi_i N_i) + (y_{11} N_5 - y_{12} N_6) \theta_{13} + (y_{21} N_5 - y_{22} N_6) \theta_{23} + (y_{32} N_6 - y_{33} N_7) \theta_{33} + (y_{43} N_7 - y_{44} N_8) \theta_{44}
\]

where \( u_1, u_2 \) and \( u_3 \) are the displacement components in the \( x, y \) and \( z \) directions respectively, of a generic point in the laminate space; \( u_{o_i}, \psi_i \), and \( w_{o_i} \) are the in-plane and transverse displacements of a point \((x, y)\) on the mid-plane respectively; \( \psi_x \) and \( \psi_y \) are the rotations of normal to the mid-plane about \( y \) and \( x \) axes respectively; \( \theta_{ij} \) (\( i=1, 4 \)) are the drilling rotations; \( N_i \) (\( i=5, 8 \)) are the shape functions; \( x_{ij} = x_i - x_j \) and \( y_{ij} = y_i - y_j \) are corner coordinate differences.

The shape functions \( N_i \) (\( i=5, 8 \)) associated with drilling rotations [18] are expressed as:

\[
  N_5 = \frac{1}{16} (1 - \xi^2) (1 - \eta) \quad N_6 = \frac{1}{16} (1 + \xi)(1 - \eta^2) \quad N_7 = \frac{1}{16} (1 - \xi^2)(1 + \eta) \quad N_8 = \frac{1}{16} (1 - \xi)(1 - \eta^2)
\]

where, \( \xi \) and \( \eta \) are the usual iso-parametric coordinates.

The strains in small deflection linear domain associated with the displacement field in Eq. (1) are

\[
  \epsilon_1 = \epsilon_1^o + z \kappa_1^o \quad \epsilon_2 = \epsilon_2^o + z \kappa_2^o \quad \epsilon_3 = 0 \quad \epsilon_4 = \epsilon_4^o \quad \epsilon_5 = \epsilon_5^o \quad \epsilon_6 = \epsilon_6^o + z \kappa_6^o
\]

where

\[
  \epsilon^o = (\epsilon_{1o}, \epsilon_{2o}, \epsilon_{6o})^T = \begin{bmatrix} \frac{\partial u_{do}}{\partial x} & \frac{\partial v_{do}}{\partial y} & \frac{\partial u_{do}}{\partial y} + \frac{\partial v_{do}}{\partial x} \end{bmatrix}^T
\]
\[ u_{do} = \sum_{i=1}^{4} \left( u_{oi} N_i \right) + \left( y_{14} N_8 - y_{21} N_8 \right) \theta_{1} + \left( y_{21} N_8 - y_{32} N_8 \right) \theta_{2} + \left( y_{32} N_8 - y_{43} N_7 \right) \theta_{3} + \left( y_{43} N_7 - y_{41} N_8 \right) \theta_{4} \]

\[ v_{do} = \sum_{i=1}^{4} \left( v_{oi} N_i \right) + \left( x_{14} N_8 - x_{12} N_8 \right) \theta_{1} + \left( x_{12} N_8 - x_{23} N_8 \right) \theta_{2} + \left( x_{23} N_8 - x_{34} N_7 \right) \theta_{3} + \left( x_{34} N_7 - x_{41} N_8 \right) \theta_{4} \]

\[ \kappa^o = \left( \kappa_1^o, \kappa_2^o, \kappa_6^o \right)^T = \left( \frac{\partial \psi_x}{\partial x} + \frac{\partial \psi_y}{\partial y} + \frac{\partial \psi_z}{\partial z} \right)^T \]

As seen from Eqs. (2) and (3), the transverse shear strains are constant through the laminate thickness as a result of which there is need for the shear correction factors [10-15].

The generalized mid-surface strains at any point given by Eq. (3) can be expressed in terms of nodal displacements \( \left( \delta_{ij} \right)_{(c)} \) as follows:

\[ \left( \varepsilon^{(c)} \right) = \left[ B_e^o \right] \left( \delta_{ij} \right)_{(c)} \quad \kappa^{o(c)} = \left[ B_e^o \right] \left( \delta_{ij} \right)_{(c)} \quad \varepsilon^{(c)} = \left[ B_e \right] \left( \delta_{ij} \right)_{(c)} \]

where \( \left[ B_e^o \right] \), \( \left[ B_e^c \right] \) and \( \left[ B_e \right] \) are generated strain-displacement matrices. One basic problem inherent in the use of standard interpolation of the strains for the transverse shear components is that the element locks when it is thin. The reason for this locking is that the element, when loaded in pure bending, will exhibit spurious transverse shear energy. In order to overcome the shear locking, Dvorkin and Bathe [19] proposed assumed interpolations for the shear strain to develop a four node assumed strain degenerated plate element [20].

The substitute shear strain fields are chosen as follows:

\[ \bar{\varepsilon}_{\xi^i}^o = \sum_{i=1}^{4} \sum_{j=1}^{2} P_i(\xi) Q_j(\eta) \varepsilon_{\xi^i}^{ij} \]

\[ \bar{\varepsilon}_{\eta^i}^o = \sum_{i=1}^{21} \sum_{j=1}^{4} Q_i(\xi) P_j(\eta) \varepsilon_{\eta^i}^{ij} \]

where, \( Q_1(\xi) = (1 + z)/2 \), \( Q_2(\xi) = (1 - z)/2 \) and \( P_i(\eta) = \{ z \} \) in which \( \varepsilon_{\xi^i}^{ij} \) and \( \varepsilon_{\eta^i}^{ij} \) are the \( m \times n \) unknown substitute shear strain parameters associated with two sets of \( m \times n \) sampling points \( \left( \tilde{\xi}_i, \tilde{\eta}_j \right) \) and \( \left( \hat{\xi}_i, \hat{\eta}_j \right) \).

In order to eliminate locking, the following equations are obtained:

\[ \bar{\varepsilon}_{\xi^i}^o \left( \tilde{\xi}_i, \tilde{\eta}_j \right) = \varepsilon_{\xi^i}^{o(c)} \left( \tilde{\xi}_i, \tilde{\eta}_j \right) \quad \left( i = 1, \ldots, m; \ j = 1, \ldots, n \right) \]

\[ \bar{\varepsilon}_{\eta^i}^o \left( \hat{\xi}_i, \hat{\eta}_j \right) = \varepsilon_{\eta^i}^{o(c)} \left( \hat{\xi}_i, \hat{\eta}_j \right) \quad \left( i = 1, \ldots, n; \ j = 1, \ldots, m \right) \]

It is possible to write

\[ \bar{\varepsilon}^o = \begin{bmatrix} \bar{\varepsilon}_{\xi}^o \\ \bar{\varepsilon}_{\eta}^o \end{bmatrix} \]

where, \( \bar{\varepsilon}_{\xi}^o \) and \( \bar{\varepsilon}_{\eta}^o \) are obtained from \( \bar{\varepsilon}_{\xi^i}^o \) and \( \bar{\varepsilon}_{\eta^i}^o \) given by Eqs. (5) and (6) by tensor transformation. The transformation of the strain tensor in curvilinear coordinates may be written as

\[ e_{\alpha\beta} = \frac{\partial \varepsilon^o}{\partial \alpha} \frac{\partial \varepsilon^o}{\partial \beta} \varepsilon_{ij} \]

where, it is assumed that \( e_{\alpha\beta} \) is the strain tensor in the \( (\xi, \eta) \) coordinate system and \( e_{ij} \) is the strain tensor in the \( (x, y) \) system [19]. For implementation purpose, \( e^o \) in Eq. (4) is replaced by \( \bar{e}^o \) where \( \bar{e}^o \) is the substitute shear strains to remove spurious zero energy modes. Hence the substitute shear strain \( \bar{e}^o \) is given by

\[ \bar{e}^{o(c)} = \left[ B_e \right] \left( \delta_{ij} \right)_{(c)} \]
where, $[B^T]^{c}$ is generated strain displacement matrix.

For arbitrary values of virtual displacements, the following assembled equation for transient analysis is stated as:

$$[M][\Delta]+[K][\Delta] = \{F\}$$

(12)

Here the unknown vector $[\Delta]$ is generated by the assemblage of element degrees of freedom $[d]^T$, $e=1,2,...$ total degrees of freedom in the region $R$. The assembled stiffness and mass for transient analysis are

$$[K]=\sum_{e} \left[ \int_{A_e} \begin{bmatrix} B^T_e AB''_e + B^T_e BB'_e + B^T_e BB'_e + B^T_e DB''_e + \bar{B}^T_e \bar{A'} \bar{B}''_e \end{bmatrix} dA \right]$$

(13)

where, $A_{ij}, B_{ij}, D_{ij}$ are the plate stiffness, defined by

$$\begin{align*}
(A_{ij}, B_{ij}, D_{ij}) &= \frac{h/2}{-h^2} [Q] (i,j = 1,2,6) \\
(A_{ij}) &= \frac{h/2}{-h^2} [Q] K_2^2 (i,j = 5,4)
\end{align*}$$

(14)

where, $K_1^2$ and $K_2^2$ are the shear correction factors [16-17] calculated from the shear strain energy formulation. $\bar{Q}_{ij}$ are the transformed plane stress reduced elastic stiffness coefficients, which are given as

$$\begin{align*}
\bar{Q}_{11} &= Q_1 c^4 + 2(Q_{12} + 2Q_{66}) k^2 s^2 + Q_{22} s^4 \\
\bar{Q}_{12} &= (Q_{11} + Q_{22} - 4Q_{66}) k^2 s^2 + Q_{12} c^4 + s^4 \\
\bar{Q}_{16} &= (Q_{11} c^2 + (Q_{12} + 2Q_{66}) (k^2 - c^2) - Q_{22} s^2) x_3 \\
\bar{Q}_{22} &= Q_{11} s^4 + 2(Q_{12} + 2Q_{66}) k^2 s^2 + Q_{22} c^4 \\
\bar{Q}_{26} &= (Q_{11} s^2 + (Q_{12} + 2Q_{66}) (c^2 - s^2) - Q_{22} c^2) x_3 \\
\bar{Q}_{66} &= (Q_{11} + Q_{22} - 2Q_{12}) k^2 s^2 + Q_{66} (c^2 - s^2)^2 \\
\bar{Q}_{55} &= c^2 Q_{55} + s^2 Q_{44} \\
\bar{Q}_{44} &= (c^2 Q_{44} + s^2 Q_{55}) \\
\bar{Q}_{45} &= cs(Q_{44} - Q_{45}) \\
\bar{Q}_{54} &= cs(Q_{55} - Q_{44})
\end{align*}$$

(15)

where, $[Q]_{ik}$ is constitutive matrix at lamina level; $c = \cos \theta$; $s = \sin \theta$. $\theta$ is the angle between the lamina $x$ axis and lamina principal $x$, $x_{ij}$, axis; with $Q_{11} = E_{1}(1-v_{12}v_{21})$, $Q_{12} = v_{12}E_{2}(1-v_{12}v_{21})$, $Q_{22} = E_{2}(1-v_{12}v_{21})$, $Q_{66} = G_{12}$, $Q_{55} = G_{13}$, $Q_{44} = G_{23}$, $E_i (i = 1,2)$ are the Young’s moduli, $G_{12}$, $G_{13}$ and $G_{23}$ are the shear moduli, $v_{12}$ and $v_{21}$ are the Poisson’s ratios.

The consistent mass matrix $[M]$ in Eq. [12] can be obtained from the kinetic energy of the system

$$\left[ \int_{V} \rho(\vec{u}_1 \vec{\alpha}_1 + \vec{u}_2 \vec{\alpha}_2 + \vec{u}_3 \vec{\alpha}_3) dV \right]$$

(16)

where $\rho(x,y,z) \text{ and } V$ are the density of the shell at $(x,y,z)$ and volume of the shell respectively.

A 2x2 Gauss-Legendre rule (i.e., full integration scheme) is employed to integrate bending, membrane, shear and inertia terms in the energy expressions for the four node drilling degrees of freedom plate element. The developed finite element should pass the pathological tests [21, 22] so that these could be used with confidence.

Numerical results are presented to assess the behaviour of a family of plate bending elements (FSDTC4, FSDTV4 and ECPT4) based on a unified first order shear deformation theory. FSCTC4, FSDTV4 and ECPT4 denote 4-node plate bending elements based on a first-order shear deformation theory with the shear correction factor of 5/6 (FSCTC) [13, 14], first order shear deformation theory with the shear correction factor from strain energy [15] formulation (FSDTV) and equivalent classical plate theory (ECPT) with the use of empirical shear correction factor derived from patch test considerations respectively. It is well known that the CPT under-predicts the deflection than shear deformable plate theories for thick laminates [13, 14]. It is found that the deflection and frequency results [13-14] from FSDT are functions of shear correction factors and width to thickness ratios of laminates and CPT results. Hence in the present analysis, equivalent classical plate theory (ECPT) results are obtained with the use of empirical shear correction factors which are found to be functions of
Reissner’s homogeneous shear correction factors (5/6) and side to thickness ratio (a/h).

In order to confirm the convergence characteristics of FSDTC4, FSDTV4 and ECPT4, a patch test is performed on plates with various mesh patterns which can represent constant moment in plates. Note that for isotropic plates, FSDTC4 and FSDTV4 give identical results. The rectangular plate in consideration is simply supported at three corners (i.e w = 0) and loaded with a concentrated load of 2 at the fourth corner and moments applied at the boundary as shown in Fig. 2.

The geometrical and material properties with consistent units are: a = 20.0, b = 10.0, h = variable, E =2.1x10^6, Poisson ratio=0.30. In order to establish the benchmark for comparison, widely used DKT plate bending element [23] is implemented presently. The mesh patterns employed for rectangular and triangular plate bending elements are shown in Fig. 2. The results from HSDT4, FSDTC4 and ECPT4 are compared with DKT in Table 1. From the results, it is observed that all the plate models compare well with each other for width to thickness ratio of 10^4 which is also reported in [19].

From numerical experiments, it is found that a value of shear correction factor

\[ K_{thin} = \frac{K_c^2}{\text{thin parameter}} 10^{-6(K_c^2 - \log(10(\text{thin parameter})))} \]

where, \( K_c^2 = \frac{5}{6} \), thin parameter = length/thickness ratio with FSDT4 gives close results with respect to DKT results which we will denote as ECPT4 (Equivalent classical four node plate element). The comparative results from ECPT4 are also shown in Table 1. As seen from the results, ECPT4 compares very well with DKT results for a wide range of width to thickness ratios.

![Fig. 2 Mesh patterns for (a) triangular and (b) rectangular elements, the boundary loads (Pz; Mx; My) are indicated in the brackets [24]](image)

**Table 1 Vertical corner displacement of a rectangular plate under constant moment patch test [24]**

<table>
<thead>
<tr>
<th>b/h</th>
<th>DKT</th>
<th>ECPT4</th>
<th>FSDT4</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td>0.185514*</td>
<td>0.185517</td>
<td>0.186534</td>
</tr>
<tr>
<td>10</td>
<td>0.00148571</td>
<td>0.00148574</td>
<td>0.00150764</td>
</tr>
<tr>
<td>20</td>
<td>0.0118857</td>
<td>0.0118859</td>
<td>0.0119296</td>
</tr>
<tr>
<td>30</td>
<td>0.0401143</td>
<td>0.0401149</td>
<td>0.0401801</td>
</tr>
<tr>
<td>40</td>
<td>0.0950857</td>
<td>0.0950871</td>
<td>0.0951735</td>
</tr>
<tr>
<td>50</td>
<td>0.185714</td>
<td>0.185717</td>
<td>0.185824</td>
</tr>
<tr>
<td>10^3</td>
<td>1.48571</td>
<td>1.48574</td>
<td>1.48939</td>
</tr>
<tr>
<td>10^4</td>
<td>1.48571</td>
<td>1.48574</td>
<td>1.48572</td>
</tr>
<tr>
<td>10^5</td>
<td>0.148571**</td>
<td>0.148589</td>
<td>0.148571</td>
</tr>
</tbody>
</table>

*Multiplied by 10^-2 ** multiplied by 10^-7

Again the static and free vibration analysis of simply supported laminated composite (0/90) plates with the following
material and geometric properties: E_1/E_2 = 25, G_{12}/E_2 = 0.5, G_{23}/E_2 = 0.20, a/b = 1; a mesh density of 9 nodes per quarter of the plate is employed to obtain present FEM results. The results for the central deflection \( \frac{1}{b^4} \frac{P}{E} \) from FSDTC, FSDTV, CPT, ECPT, FSDTC4, FSDTV4 and ECPT4 are shown in Table 2. As seen from the results, there is close agreement among analytical and numerical results.

IV. TWO DIMENSIONAL FE FORMULATIONS FOR SANDWICH SHELLS BASED ON FLAT-FACETTED SHELL FORMULATION

The procedures of deriving the shell equations based on flat-facetted formulations are the same as that of the previous section. Only the transformation matrix [1] will be used to convert stiffness, mass matrices and force vectors from local to global coordinates.

By the rules of orthogonal transformation the stiffness, mass and buckling matrices of an element in global co-ordinate become

\[
[K]^g = T_g^T [K] T_g
\]
\[
[M]^g = T_g^T [M] T_g
\]

where, \( T_g \) is the transformation matrix from local to global axes as given below

\[
T_g = \begin{bmatrix}
T_{g\text{top}} & 0 \\
0 & T_{g\text{bot}}
\end{bmatrix}
\]
\[
T_{g\text{top}} = \begin{bmatrix}
\cos(X,x) & \cos(Y,x) & \cos(Z,x) \\
\cos(X,y) & \cos(Y,y) & \cos(Z,y) \\
\cos(X,z) & \cos(Y,z) & \cos(Z,z)
\end{bmatrix}
\]
\[
T_{g\text{bot}} = \begin{bmatrix}
\cos(Y,y) & -\cos(Y,x) & \cos(Y,z) \\
-\cos(X,y) & \cos(X,x) & -\cos(X,z) \\
\cos(Z,y) & -\cos(Z,x) & \cos(Z,z)
\end{bmatrix}
\]

and \((X,x)\) denotes the angle between the positive X (global) and \(x\) (local) axes.

V. TWO DIMENSIONAL FE FORMULATIONS FOR SANDWICH SHELLS BASED ON FLAT-FACETTED SHELL FORMULATION

A composite doubly curved composite sandwich shell as shown in Fig. 3 is represented by the curvilinear dimensional coordinates \(x\) and \(y\) that coincides with the mid-surface of the shell and \(z\)-axis is oriented in the thickness direction. The displacement components of a generic point in the shell are assumed as given by Eq. (1).

Fig. 3 Doubly curved Laminate geometry with positive set of lamina/laminate reference axes, displacement components and fibre orientation
The strains in small deflection linear domain associated with the displacement field in Eq. (1) in the doubly curved shell formulation are

\[
\begin{align*}
\varepsilon_{xx} &= \varepsilon_{xx}^o + \kappa \kappa_{xx} + \frac{w_o}{R_1} \\
\varepsilon_{yy} &= \varepsilon_{yy}^o + \kappa \kappa_{yy} + \frac{w_o}{R_2} \\
\varepsilon_{zz} &= 0 \\
\varepsilon_{yz} &= \varepsilon_{yz}^o - \frac{v_o}{R_2} - \frac{\psi_y}{R_2} \\
\varepsilon_{zx} &= \varepsilon_{zx}^o - \frac{u_o}{R_1} - \frac{\psi_x}{R_1} \\
\varepsilon_{xy} &= \varepsilon_{xy}^o + \kappa \kappa_{xy}
\end{align*}
\]  

(20)

where, \( R_1 \) and \( R_2 \) are the principal radii of curvature of the middle surface [7]. For plates, both radii of curvature have a very high value close to infinity (1/R1=0 and 1/R2=0). The other notations used in Eq. (20) are defined by Eq. (3). As usual, Hamilton’s principle [5] is used to derive the equations of motion appropriate for the displacement field in Eq. (1), the strain displacement Eq. (20) and the constitutive relations in Eqs. (4) to (8) to represent Eq. (9). The membrane, bending and shear strains for doubly curved shell formulation are represented by

\[
\varepsilon_m = \begin{bmatrix} \frac{\partial u_o}{\partial x} + \frac{w_o}{R_1} \\ \frac{\partial v_o}{\partial y} + \frac{w_o}{R_2} \\ \frac{\partial v_o}{\partial x} + \frac{\partial u_o}{\partial y} \end{bmatrix}, \quad \varepsilon_b = \begin{bmatrix} \frac{\partial \psi_x}{\partial x} \\ \frac{\partial \psi_y}{\partial y} \\ \frac{\partial \psi_y}{\partial x} + \frac{\partial \psi_x}{\partial y} \end{bmatrix}, \quad \varepsilon_s = \begin{bmatrix} \frac{\partial \psi_o}{\partial x} - \frac{u_o}{R_1} \\ \frac{\partial \psi_o}{\partial y} - \frac{v_o}{R_2} \\ \psi_2 \end{bmatrix}
\]  

(21)

The stress resultant vector \([N_{xx}, N_{yy}, N_{xy}, M_{xx}, M_{yy}, M_{xy}, Q_{xz}, Q_{yz}, S_{xz}, S_{yz}]\) are given by the following equations

\[
\begin{bmatrix} N_{xx} \\ N_{yy} \\ N_{xy} \\ M_{xx} \\ M_{yy} \\ M_{xy} \\ Q_{xz} \\ Q_{yz} \\ S_{xz} \\ S_{yz} \end{bmatrix} = \int_0^h \begin{bmatrix} \sigma_{xx} \\ \sigma_{yy} \\ \sigma_{xy} \end{bmatrix} [l, z] \, dz
\]  

(22)

The laminate constitutive relations are obtained as
The virtual work equation can be written in compact form as

$$
\int_A \left( \delta \varepsilon_m^T A \varepsilon_m + \delta \varepsilon_b^T B \varepsilon_b + \delta \varepsilon_b^T B \varepsilon_m + \delta \varepsilon_b^T D \varepsilon_b + \delta \varepsilon_s^T A' \varepsilon_s \right) dA + \int_A \int_A q d \\ v dA d t + \int_A \int_A I_1 (u \delta u + v \delta v + w \delta w) + I_2 (\psi \delta \psi + \phi \delta \phi + \xi \delta \xi + \eta \delta \eta) dA d t
$$

Once the virtual work equation is known, the finite element models that pass the benchmark tests [21, 22] can be readily obtained by following the steps as mentioned earlier by Eqs. (15) to (19).

VI. TWO DIMENSIONAL FE FORMULATIONS FOR SANDWICH SHELLS
BASED ON THREE DIMENSION DEGENERATED SHELL FORMULATION

In this approach, the three dimensional stress and strain conditions are degenerated to shell behaviour. The definition of independent rotational and displacement degrees of freedom permits transverse shear deformation to be taken into account, since rotations are not tied to the slope of the mid-surface. This approach is equivalent to using a general shell theory and reduces to the Eq. (1) when applied to the plates. A typical quadrilateral degenerated shell element is shown in Fig. 4. Also, triangular degenerated shell elements can be represented similar to Fig. 4. The displacement components of the mid-point of the normals, the nodal coordinates, global stiffness matrices, applied force vectors etc. are referred the global coordinate system \((x, y, z)\). At \(k\)th node, \(\vec{V}_{3k}\) is constructed as a vector joining the top and bottom of the node in nodal coordinate system \(\vec{V}_{1k}, \vec{V}_{2k}, \vec{V}_{3k}\). \(\vec{V}_{1k}\) is constructed parallel to the global \(x z\) plane or is assumed parallel to \(x\) axis when \(\vec{V}_{3k}\) is in \(y\) direction. And consequently \(\vec{V}_{2k}\) is derived as cross products of \(\vec{V}_{3k}\) and \(\vec{V}_{1k} \cdot \xi - \eta - \zeta\) is a natural coordinate system; \(\xi\) and \(\eta\) are the curvilinear coordinates at the middle surface. \(\zeta\) is linear coordinate in thickness direction with \(\zeta = +1\) and \(-1\) at top and bottom surfaces, respectively.

![Fig. 4 Degenerated quadrilateral shell element with various co-ordinate systems](image-url)

The position of an arbitrary point of the shell [1] is obtained as

$$
\begin{bmatrix}
    N \\
    M \\
    Q
\end{bmatrix} =
\begin{bmatrix}
    A & B & 0 \\
    B & D & 0 \\
    0 & 0 & A'
\end{bmatrix}
\begin{bmatrix}
    \varepsilon_m \\
    \varepsilon_b \\
    \varepsilon_s
\end{bmatrix}
$$

(23)

The virtual work equation can be written in compact form as

$$
\int_A \left( \delta \varepsilon_m^T A \varepsilon_m + \delta \varepsilon_b^T B \varepsilon_b + \delta \varepsilon_b^T B \varepsilon_m + \delta \varepsilon_b^T D \varepsilon_b + \delta \varepsilon_s^T A' \varepsilon_s \right) dA + \int_A \int_A q d \\ v dA d t + \int_A \int_A I_1 (u \delta u + v \delta v + w \delta w) + I_2 (\psi \delta \psi + \phi \delta \phi + \xi \delta \xi + \eta \delta \eta) dA d t
$$

(24)

Once the virtual work equation is known, the finite element models that pass the benchmark tests [21, 22] can be readily obtained by following the steps as mentioned earlier by Eqs. (15) to (19).

VI. TWO DIMENSIONAL FE FORMULATIONS FOR SANDWICH SHELLS
BASED ON THREE DIMENSION DEGENERATED SHELL FORMULATION

In this approach, the three dimensional stress and strain conditions are degenerated to shell behaviour. The definition of independent rotational and displacement degrees of freedom permits transverse shear deformation to be taken into account, since rotations are not tied to the slope of the mid-surface. This approach is equivalent to using a general shell theory and reduces to the Eq. (1) when applied to the plates. A typical quadrilateral degenerated shell element is shown in Fig. 4. Also, triangular degenerated shell elements can be represented similar to Fig. 4. The displacement components of the mid-point of the normals, the nodal coordinates, global stiffness matrices, applied force vectors etc. are referred the global coordinate system \((x, y, z)\). At \(k\)th node, \(\vec{V}_{3k}\) is constructed as a vector joining the top and bottom of the node in nodal coordinate system \(\vec{V}_{1k}, \vec{V}_{2k}, \vec{V}_{3k}\). \(\vec{V}_{1k}\) is constructed parallel to the global \(x z\) plane or is assumed parallel to \(x\) axis when \(\vec{V}_{3k}\) is in \(y\) direction. And consequently \(\vec{V}_{2k}\) is derived as cross products of \(\vec{V}_{3k}\) and \(\vec{V}_{1k} \cdot \xi - \eta - \zeta\) is a natural coordinate system; \(\xi\) and \(\eta\) are the curvilinear coordinates at the middle surface. \(\zeta\) is linear coordinate in thickness direction with \(\zeta = +1\) and \(-1\) at top and bottom surfaces, respectively.

![Fig. 4 Degenerated quadrilateral shell element with various co-ordinate systems](image-url)

The position of an arbitrary point of the shell [1] is obtained as

$$
\begin{bmatrix}
    x \\
    y \\
    \zeta
\end{bmatrix} =
\sum_{k=1}^{n} N_k \begin{bmatrix}
    x_k \\
    y_k \\
    \zeta_k
\end{bmatrix} + \frac{\zeta}{2} \sum_{k=1}^{n} h_k \begin{bmatrix}
    \vec{V}_{3k}^x \\
    \vec{V}_{3k}^y \\
    \vec{V}_{3k}^z
\end{bmatrix}
$$

(25)
where, $\hat{V}_{ik}^j$ ($i=1,2,3$) is the $j^{th}$ component of unit vector along nodal vector $\hat{V}_{ik}$ at node $k$ and $h_k$ is the thickness of shell at node $k$ and $n$ is the number of nodes. $x^k_o$, $y^k_o$ and $z^k_o$ are the Cartesian coordinates of the mid-point of the shell at $k^{th}$ node. The global displacement components of any point in the shell global coordinate system in terms of thickness coordinate $z$ is given by

$$
u_i = u^k_{oi} + \hat{V}_{ik}^j \delta_j + \hat{V}_{jk}^i \delta_2 \ (i=1,2,3)$$

(26)

where $\nu_i$ ($i=1,2,3$) are the displacements $u$, $v$ and $w$ in global coordinate $x$, $y$ and $z$ directions respectively. $u^k_{oi}$ ($i=1, 2, 3$) are the displacements in the midpoint of the normal in global coordinate system.

Expanding the Eq. [26], the element displacement field is expressed as

$$\begin{pmatrix} 
\nu_i \\
v \\
w 
\end{pmatrix} = \sum_{k=1}^n N_k \begin{pmatrix} u^k_o \\
v^k_o \\
w^k_o 
\end{pmatrix} + \sum_{k=1}^n N_k \frac{h_k}{2} \begin{pmatrix} \hat{V}_{ik}^x - \hat{V}_{2k}^x \\
\hat{V}_{ik}^y - \hat{V}_{2k}^y \\
\hat{V}_{ik}^z - \hat{V}_{2k}^z 
\end{pmatrix}$$

(28)

where, $u^k_o$, $v^k_o$ and $w^k_o$ are the displacement components of the midpoint of the normal in global coordinate system.

The normal strain component in the thickness direction is neglected. Owing to this assumption, the five strain components in local coordinate system are given by

$$\begin{pmatrix} 
\varepsilon_x \\
\varepsilon_y \\
\varepsilon_z \\
\gamma_{xz} \\
\gamma_{yz} 
\end{pmatrix} = \begin{pmatrix} 
\frac{\partial u}{\partial x} \\
\frac{\partial v}{\partial y} \\
\frac{\partial w}{\partial z} \\
\frac{\partial u}{\partial z} + \frac{\partial v}{\partial x} \\
\frac{\partial w}{\partial x} + \frac{\partial u}{\partial z} 
\end{pmatrix}$$

(29)

where, $\varepsilon$ with subscripts $x$, $y$ and $z$ are the normal strains in the respective directions and with subscripts $x'y'$, $x'z'$ and $y'z'$ are the shear strains on $x'y'$, $x'z'$ and $y'z'$ planes respectively; $u'$, $v'$ and $w'$ are the displacement components in the local coordinate system.

The relation between the displacements derivatives in local and global coordinates is given by

$$\begin{pmatrix} 
\hat{u}_x \\
\hat{u}_y \\
\hat{u}_z \\
\hat{v}_x \\
\hat{v}_y \\
\hat{v}_z \\
\hat{w}_x \\
\hat{w}_y \\
\hat{w}_z 
\end{pmatrix} = \begin{pmatrix} 
\hat{u}_x \\
\hat{u}_y \\
\hat{u}_z \\
\hat{v}_x \\
\hat{v}_y \\
\hat{v}_z \\
\hat{w}_x \\
\hat{w}_y \\
\hat{w}_z 
\end{pmatrix} \begin{pmatrix} 
\frac{\partial u}{\partial x} & \frac{\partial u}{\partial y} & \frac{\partial u}{\partial z} \\
\frac{\partial v}{\partial x} & \frac{\partial v}{\partial y} & \frac{\partial v}{\partial z} \\
\frac{\partial w}{\partial x} & \frac{\partial w}{\partial y} & \frac{\partial w}{\partial z} \\
\frac{\partial u}{\partial x} & \frac{\partial u}{\partial y} & \frac{\partial u}{\partial z} \\
\frac{\partial v}{\partial x} & \frac{\partial v}{\partial y} & \frac{\partial v}{\partial z} \\
\frac{\partial w}{\partial x} & \frac{\partial w}{\partial y} & \frac{\partial w}{\partial z} \\
\frac{\partial u}{\partial x} & \frac{\partial u}{\partial y} & \frac{\partial u}{\partial z} \\
\frac{\partial v}{\partial x} & \frac{\partial v}{\partial y} & \frac{\partial v}{\partial z} \\
\frac{\partial w}{\partial x} & \frac{\partial w}{\partial y} & \frac{\partial w}{\partial z} 
\end{pmatrix}$$

$$\begin{pmatrix} 
l_1 \\
l_2 \\
l_3 
\end{pmatrix} = \begin{pmatrix} 
m_1 & m_2 & m_3 \\
n_1 & n_2 & n_3 
\end{pmatrix} \begin{pmatrix} 
l_1 \\
l_2 \\
l_3 
\end{pmatrix}$$

(30)

where $l_i$, $m_i$ and $n_i$ ($i=1,2,3$) are the components of unit vectors.

The relation between the derivatives in global coordinates and natural coordinates is expressed as
The displacement derivatives with respect to $\xi$ are given by

$$\begin{bmatrix} u_{,\xi} \\ v_{,\xi} \\ w_{,\xi} \end{bmatrix} = \sum_{k=1}^{n} N_{k,\xi} \begin{bmatrix} u_{o,k} \\ v_{o,k} \\ w_{o,k} \end{bmatrix} + \sum_{k=1}^{n} N_{k,\xi} \xi \frac{h_k}{2} \begin{bmatrix} \vec{V}_{1k} - \vec{V}_{2k} \\ \vec{V}_{1k} - \vec{V}_{2k} \\ \vec{V}_{1k} - \vec{V}_{2k} \end{bmatrix} \begin{bmatrix} \beta_{1k} \\ \beta_{2k} \end{bmatrix}$$

Similarly the displacement derivatives with respect to $\eta$ and $\zeta$ can be obtained. The strain displacement equation relating strain components in Eq. (2) in global coordinate system to the nodal variables can be obtained. The stress components can be obtained by utilising Eqs. (4) and (5). Again Hamilton’s equation Eq. (9)) and Virtual work principles (Eq. (14)) as described previously are applied to get the system stiffness, mass matrices and force vectors.

VII. THREE DIMENSIONAL FE FORMULATIONS FOR SANDWICH SHELLS BASED ON SOLID SHELL FORMULATION

The various coordinate systems as described in Fig. 4 are also used to derive solid shell finite elements. In the global coordinate, the position vector $X$ of a point inside the element can be described as:

$$X = \vec{X} + \zeta P$$

where $\vec{X}$ is the position vector for the mid-surface and $P$ is half of the vector pointing from the lower face to the upper face.

Let $X_{i}^U$ and $X_{i}^L$ ($i=1,2,3,4$) be the nodal position vectors for nodes in the upper face and lower face of the element respectively, and

$$\vec{X}_i = \frac{1}{2} \left( X_i^U + X_i^L \right) \quad P_i = \frac{1}{2} \left( X_i^U - X_i^L \right)$$

where $\vec{X}_i$ is the position vector at the node on the mid-surface and $P_i$ is half of the nodal vectors pointing from the lower nodes to the upper ones.

The position vector $X$ and the displacement vector $u$ in the form of isoparametric mapping can be expressed as

$$X = \sum_{k=1}^{n} N_k (\vec{X}_k + \zeta P_k)$$

$$u = \sum_{k=1}^{n} N_k (\vec{u}_k + \zeta p_k)$$

where

$$\vec{u}_i = \frac{1}{2} \left( u_i^U + u_i^L \right)$$

$$p_i = \frac{1}{2} \left( p_i^U + p_i^L \right)$$

Here $u_i^U$ and $u_i^L$ are nodal displacement vectors (in the local coordinate) for the upper and the lower faces, respectively.

With the definitions in Eqs. (35) and (36), the solid shell element formulations are similar to the shell elements as described previously. Hence following the usual finite element formulations, the system stiffness, mass matrices and force vectors can be obtained. It should be noted that solid finite elements contain degrees of freedom at the upper and lower faces of the laminae. Hence the laminate analysis is carried out by assembling the matrices layer by layer as opposed to two dimensional formulations described previously.
VIII. PLANE STRESS ANALYSIS FOR COMPRESSION STRENGTH OF THICK COMPOSITE STRUCTURES

The composite material usage in primary load bearing structures has led to increasing thickness of the fabricated composite parts. The manufacturing of thicker composite parts can lead to the introduction of defects during the resin infusion and curing process. Typical defects like fiber waviness, voids and delaminations are observed in such components. Understanding the compressive strength limits of UD composites and their dependency on the defect size is critical from a structural design perspective [21]. Micromechanical modeling of composite parts to predict compressive strength of a thick UD composite material is challenging due to the computational cost of modeling individual fibers and matrix layers.

Waviness can be induced in the fiber during the layup and resin infusion process or during the curing process due to thermal mismatch between laminate and the mold surface. Typically, fiber waviness will lead to local shear yielding of the matrix layers adjacent to the wavy fibers, which causes fiber microbuckling and kink-band type of failure. Another mode of failure observed under compression is by splitting at the fiber-matrix interface.

The number of experimental studies [21] on the thickness effects on compressive strength is rather limited. This is primarily due to the fact that problems related with testing, such as increased chances of premature failure due to end crushing, are exacerbated with thicker composites. In spite of such difficulties, test results have revealed the tendency that the failure strength decreases with increasing thickness of composite laminates. Hence, there is a definite need to develop reliable models that capture this thickness scaling trend while incorporating fiber waviness as the sole failure mode. This can lead to computational as well as experimental benefits in predicting compressive response of thick composites using scaled, thin coupon finite element and test geometries.

To enable a controlled study on fiber wave induced compressive failure in this work, the fiber wave has been characterized by the fiber wave length (L) and height (a), along with the coupon thickness (tc) (Fig. 5), with the fiber waves present all through the coupon thickness.

The overall coupon length was chosen appropriately - short enough that global Euler buckling would be avoided and long enough that the ends did not affect the stress state near the wavy region. In all analyses and testing carried out, it was verified that the Euler critical buckling load was much higher than the peak stress obtained for the corresponding case thus eliminating global buckling as a failure mode.

In this approach, a scaled down model of the composite with alternating layers of fiber and resin was created. The resin layers were epoxy (Fig. 6) and modeled with elastic-plastic material properties and the glass fiber was modeled using linearly elastic properties [25].

A parametric FE micromechanical model (Figs. 7 and 8) was developed in ANSYS® to model compression on a two-dimensional coupon with a given surface defect and coupon thickness. The required geometric inputs and material parameters are read in to generate the model through ANSYS® scripts.

The surface and interior defect waves are generated using a combination of a cosine function and a circular fillet that smoothly transitions the cosine curve to the straight portion of the coupon. Propagating the surface wave towards the root defect along directions obtained using the surface and root defect waves generates the intermediate layers. It is worth noting
that the intermediate layers will be of uniform thickness only for the case when this propagation direction coincides with the normal at each point on the cosine curves (of the surface and root defects). For any other direction of propagation, the layers will have non-uniform thicknesses. In order to address issue, the intermediate layers are generated as follows: (i), the centroidal line of a fictitious tow comprised of one fiber layer with half a layer of resin on either side is propagated by the aforementioned approach; (ii), fiber areas of uniform thickness are generated about each centroidal line; and (iii), the remaining areas surrounding the fibers are filled with resin. This approach ensures that the fiber layers are of uniform thickness and all non-uniformity is distributed in the resin layers, thus mimicking a practical fabrication process.

Here, a practical issue resulting from the use of fillets in the geometry is addressed. The user input wave heights (of surface and interior waves) are lowered because of fillet insertion at ends of the wave geometries. Further, the defect locations through the thickness as well as the coupon thickness are modified because of the discreteness of the number of fiber and matrix layers. The input wavelengths (of surface and interior waves) remain unchanged. In order to correct for these modifications to the desired inputs, an inverse geometry calculation is performed over a design space to find the right required input geometry values that will generate the desired user-input geometries. The user-input geometry is set as the target with respect to which the cost of deviating is minimized. This results in generating a model whose dimensions best match the user-input geometry.

A nonlinear, large deformation analysis is carried out using displacement controlled axial compressive loading applied to the end faces, each of which is coupled to a multi-point constraint (MPC) node. The peak load is captured as a nodal reaction load at the point of unloading, representing a fiber microbuckling instability. The post peak analysis is not continued, as the interest is to capture the first point of failure.

The tow-level model is one in which the fiber and resin properties are homogenized to generate effective properties of a tow layer (Fig. 9).

Comparing identical coupon and wave geometries modeled with a tow model vs. a micromechanics model as described previously leads to the following modifications:

(i) The tow has linearly elastic properties obtained from homogenizing the linearly elastic moduli of the fiber and resin.

(ii) The thickness of the tow, which is several times the fiber thickness, is computed by ensuring that the overall fiber volume fraction remains the same between both modeling approaches. One important consequence of this modification is that the effective resin thickness that can be plastically deformed reduces when compared to the micromechanics model. In general, this results in higher coupon peak compressive response than a micromechanics model of identical coupon and wave geometries.

(iii) The material properties for the tow have to be specified using local element coordinates because the homogenization creates a material directionality to the tow layer.

Thus, the tow model enables a reduced computational cost arising from discretization, model size and analysis time. However, this computational advantage also results in the loss of some stress state detail within the tow (i.e., in the individual fibers and resin comprising the tow) as well as the overhead of maintaining local element coordinates to specify material properties for the tow. The material properties of the tow layer as well as the creation of the FE model including the local element coordinates are shown in Figs. 10 and 11.

![Fig. 9 Schematic cross section of a tow layer embedded in resin.](image1)

![Fig. 10 FE Tow level model of the coupon](image2)

![Fig. 11 View of elemental coordinate systems in the FE Tow level model of the Coupon](image3)
The micromechanical model was evaluated against a physical coupon test and the tow model for peak compressive coupon response. The parameterized micromechanical model was run for different coupon thicknesses while maintaining the other non-dimensional geometric parameters, i.e., the wave aspect ratio \((aR = L/a)\) and the wave height as a percentage of the thickness \((wH = a/tc)\), constant. The peak compressive response is compared between the micromechanics analyses and the physical test in Fig. 12. A tapering plateau effect of the compressive response with respect to coupon thickness is immediately apparent from the micromechanical analysis. The fluctuations in micromechanical peak stress with respect to thicknesses less than 4 mm are not considered to be significant. However, the overall trend of peak stress tapering to a plateau with increasing coupon thickness still remains. Further, beyond around 6 mm of coupon thickness, the micromechanics results are in excellent agreement with the tests (performed on a coupon 20 mm thick). The test coupons were made from a panel and the wave defect height and other parameters were measured at five different locations for each panel. The variability in the various defect geometric parameters leads to variability in the failure strength of the composites. Another component adding to variability in the failure strength of tested coupons is the material property variation of the resin. The good agreement between the micromechanical kink-band model and the experimental test indicates that the first peak stress is largely governed by kink-band formation as the failure mechanism.

Next, the micromechanical model was evaluated against the tow model for a specific wave geometry. The peak compressive response for increasing coupon thicknesses in the micromechanics model was compared (Fig. 13) with the tow model results for a 20 mm thick coupon. As discussed previously, the peak compressive response from the tow model upper-bounds the converged (with respect to coupon thickness) micromechanics response. This brings to attention that the strain levels in the resin within the tow reach yield values, which are not captured by the homogenized, linearly elastic tow material properties. It provides further support to the fact that the micromechanics model affords a level of detail that is both necessary and sufficient - necessary because of the results in Fig. 13, and sufficient because of the good agreement with the coupon test in Fig. 12.

Further, the micromechanical model was compared with the tow model for different wave geometries for a fixed wave height (Fig. 14). It is evident that a 6 mm thick micromechanics model sufficiently captures the compressive peak response as provided by a 20 mm thick tow model for a variety of wave geometries.
Finally, the thickness variation in the micromechanics model is explored in the case of severe defect geometry with steep fiber waves (Fig. 15). The conclusions from Figs. 12 and 13 on the thickness variation of the compressive response remain applicable to this case too, and a tapering trend of the compressive response with respect to coupon thickness is observed once again.

**IX. PLANE STRAIN ANALYSIS FOR TENSILE STRENGTH OF T-COMPOSITE JOINTS**

T joints form parts of composite wind turbine blades. Hence MSC/PATRAN 2005 was used as the pre-processor for the T joint analysis [26] as shown in Fig. 16.
For the simulations and post-processing steps, ABAQUS 6.7 was used. The details of a 3D model was prepared in PATRAN as shown in Fig. 17.

The T joint was reduced to a 2D model in the x-y plane under plane strain for initial analysis. Local rectangular coordinate systems were implemented in the structural parts so as to assign material properties accordingly. A cylindrical coordinate system was applied for the materials in the curved region of the T-Joint. For initial analysis, three nodded triangular elements are used. The coordinates of all points as shown in Fig. 17 were calculated by implementing sub-elements of T Joint in ABAQUS/CAE. The numerical values are then used in PATRAN to create 3D and 2D models. It was decided to assume plane stress conditions followed by plane strain conditions to understand the behaviour of T-Joint qualitatively. The following material properties are used for the materials used in the T-Joint.

- **For 0/90 Woven Roving Lamina:**
  \[ E_{11} = 23.47 \text{ GPa}, \quad E_{22} = 23.56 \text{ GPa}, \quad E_{33} = 10.45 \text{ GPa}, \quad G_{12} = 3.41 \text{ GPa}, \quad G_{13} = 3.25 \text{ GPa}, \quad G_{23} = 3.25 \text{ GPa}, \quad \nu_{12} = 0.128, \quad \nu_{13} = 0.406, \quad \nu_{23} = 0.406. \]

- **For +/- 45 ST Lamina:**
  \[ E_{11} = 16.96 \text{ GPa}, \quad E_{22} = 23.56 \text{ GPa}, \quad E_{33} = 16.96 \text{ GPa}, \quad G_{12} = 3.33 \text{ GPa}, \quad G_{13} = 3.25 \text{ GPa}, \quad G_{23} = 3.33 \text{ GPa}, \quad \nu_{12} = 0.154, \quad \nu_{13} = 0.293, \quad \nu_{23} = 0.267. \]

- **For Balsa:**
  \[ E_{11} = 3.355 \text{ GPa}, \quad E_{22} = 0.054 \text{ GPa}, \quad E_{33} = 0.054 \text{ GPa}, \quad G_{12} = 0.201 \text{ GPa}, \quad G_{13} = 0.201 \text{ GPa}, \quad G_{23} = 0.059 \text{ GPa}, \quad \nu_{12} = 0.351, \quad \nu_{13} = 0.351, \quad \nu_{23} = 0.360. \]

The details of the behaviour of the T-joint under 0.01 inch displacement at the center are shown in Figs. 18 and 19 for plane strain conditions.
Though plane stress and plane strain elements give stress distributions in joints, it is not possible to obtain all the components of strain and stress distributions in a joint using plane stress and plane strain elements. Hence generalized plane strain analysis is carried out to get the results for three dimensional stress and strain distributions in a laminate. In order to test the accuracy of generalized plane strain elements, two patch tests are performed on isotropic and anisotropic materials. The patch size is \( a=1\text{mm}, b=1\text{mm}, h=1\text{mm} \) as shown in Fig. 20.
The following material properties are selected: For isotropic cases: \( E = 210.0 \text{ GPa} \) and \( \nu = 0.30 \) IM7/8552 unidirectional graphite/epoxy prepreg [27]. \( E_{x1} = 161.0 \) GPa, \( E_{x2} = E_{x3} = 11.38 \) GPa, \( G_{12} = G_{13} = 5.17 \) GPa, \( \nu_{12} = \nu_{13} = 0.32 \), \( G_{23} = 3.92 \) GPa, \( \nu_{23} = 0.45 \). Analysis is performed on CMAP [26] (a composite material application program based on closed form solution) to get the information about the strains to be applied to the considered problem. A load of 2100 MPa is applied in \( x \)-direction which generates a strain of 0.01 in \( x \)-direction and -0.0030 in \( y \)-direction. As expected the strain in \( z \)-direction is \( \varepsilon_{zz} = -\nu_{xy} \varepsilon_{xx} \). A generalized plane strain element (CPEG3N) [28] is implemented to generate the test problem for ABAQUS element CPEG3. Details of the generalized plane strain finite element formulation can be found in [28]. It was found that ABAQUS CPEG3 element does not pass the required test for anisotropic cases. A problem is undertaken for a laminated \((0/45/-45/90)\) plate under tension in \( x \)-direction. \( a=1\text{mm}, b=267.2\text{mm}, h=10.688\text{mm}, b/h=25, h_{\text{ply}}=1.336\text{mm} \). The model is created in ABAQUS as shown in Fig. 21 to generate the input file for CPEG3N.

The results for stresses and strain are compared with CMAP developed by UD-CCM. Details are shown in Figs. 22-34.
Recent Advances in Composite Materials for Wind Turbine Blades

Fig. 23 Test problem for laminate [0/45/-45/90]_4 free edge problem, Variation of $\sigma_{zz}$ stress through the width

Fig. 24 Test problem for laminate [0/45/-45/90]_4 free edge problem, Variation of $\sigma_{zx}$ stress through the thickness

Fig. 25 Test problem for laminate [0/45/-45/90]_4 free edge problem, Variation of $\sigma_{xx}$ stress through the width
Finite Element Modeling of Composite Wind Turbine Blades

Fig. 26 Test problem for laminate \([0/45/-45/90]_S\) free edge problem, Variation of \(\sigma_{yz}\) stress through the width

Fig. 27 Test problem for laminate \([0/90]_S\) free edge problem, Variation of \(\sigma_{zz}\) stress through the width

Fig. 28 Test problem for laminate \([0/90]_S\) free edge problem, Variation of \(\sigma_{yz}\) stress through the width
Recent Advances in Composite Materials for Wind Turbine Blades

Fig. 29 Test problem for laminate [0/90], free edge problem, Variation of $v(y,2h)$ through the width $Y/B$

![Graph showing variation of $v(y,2h)$](image)

Fig. 30 Test problem for laminate [0/90], free edge problem, Variation of $\sigma_{zz}$ stress through the thickness $Z/H$

![Graph showing variation of $\sigma_{zz}$](image)

Fig. 31 Test problem for laminate [0/90], free edge problem, Variation of stresses and strains through the thickness $z/h$

![Graphs showing variation of stresses and strains](image)
Finite Element Modeling of Composite Wind Turbine Blades

Fig. 32 Test problem for laminate [0]_4, free edge problem, Variation of $\sigma_{zz}$ stress through the thickness $Z/H$

Fig. 33 Test problem for laminate [0]_4, free edge problem, Variation of $v(y, 2h)$ through the width $Y/B$

Fig. 34 Test problem for laminate [0]_4, free edge problem, Variation of $\sigma_{yz}$ stress through the width $Y/B$

The details of other finite elements for composite panels which find applications in composite wind turbine blades under varying loading conditions can be found in [29-38].

XI. CONCLUSIONS

In this chapter, various formulations in the analysis of composite plates and shells are described. Plane stress, plane strain and generalised plane strain finite element approaches to deal with composite materials which found applications in composite
wind turbine applications are discussed. The following specific conclusions can be made:

- Two dimensional finite element formulations for composite sandwich plates are presented. A plate bending element based on first order shear deformation theory (FSDT) is developed. A drilling rotational degree of freedom is implemented within the plate finite element formulation. Assumed strain concept is adopted for a defect free finite element. Widely used discrete Kirchhoff theory (DKT) plate bending element based on classical laminate plate theory (CLPT) is implemented for a patch test problem. An empirical shear correction factor which is a function of Reissner’s classic shear correction factor of 5/6 and thin parameter involving length to thickness ratios is proposed. It is found that the results from FSDT with the use of empirical shear correction factor gives close results with respect to CLPT. Hence it could be stated that the CLPT is a subset of FSDT with the use of appropriate shear correction factor.

- Two dimensional assumed strain finite element formulations for sandwich shells based on flat faceted shell formulation are developed. Transformation matrix from local to global axes is used for obtaining stiffness and consistent mass matrices. It is found that the shell finite element with incorporation of drilling rotational degree of freedom passes the obstacle tests and gives satisfactory results on a wide range of static and dynamic field problems.

- Two dimensional finite element formulation based on doubly curved shell formulation is presented. Two dimensional finite element formulations for sandwich shells based on three dimensional degenerated shell formulation is discussed. Since for very thick plates and shells, two dimensional formulations give unsatisfactory results, a three dimensional FE formulations for sandwich shells based on solid shell formulation is given.

- The compressive peak response of a coupon with fiber waviness is studied for different wave geometries and coupon thicknesses. Two modelling approaches are explored - (i) a micromechanics approach in which individual fiber and resin layers are explicitly modelled, and (ii) a tow-level approach in which the fiber and resin properties are homogenized to generate effective properties of a tow which is comprised of a fibers and resin. The following points are in order: a) The micromechanics model is in excellent agreement with the coupon test confirming that first failure is kink-band dominated. b) The tow model provides an upper-bound to the peak compressive response generated from the micromechanics model. It is hypothesized that this is caused by the tow properties being elastic whereas the micromechanics includes the complete elastic-plastic detail of the resin within the tow thereby capturing the material behaviour more accurately. c) In both the above comparisons, the peak compressive response from the micromechanics model is studied for increasing coupon thicknesses and a tapering trend converging to a plateau value was observed. d) A 6 mm thick micromechanics model is able to capture the peak compressive response of a 20 mm thick tow model for a variety of fiber wave geometries, thereby lending support to the fact that an appropriately scaled micromechanics model is capable of capturing bulk response for thick geometries accurately, while simultaneously including individual fiber and resin detail.

- A plane strain finite element formulation for tensile strength of T composite joints has been discussed. Different coordinate systems have been used to carry out the analysis. Though it is possible to get two dimensional stress and strain parameters within a plane strain formulation, it is not possible to get all three dimensional quantities required for complete study. Hence a generalised plane strain formulation is discussed where it is possible to get all the three dimensional quantities.

It is hoped that all finite element formulations discussed above are essential to model the composite wind turbine blades for complex design issues.

REFERENCES


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This book of science and technology provides an overview of recent research activities on the application of fibre-reinforced composite materials used in wind turbine blades. Great emphasis was given to the work of scientists, researchers and industrialists who are active in the field and to the latest developments achieved in new materials, manufacturing processes, architectures, aerodynamics, optimum design, testing techniques, etc.. These innovative topics will open up great perspectives for the development of large scale blades for on- and off-shore applications. In addition, the variety of the presented chapters will offer readers access to global studies of research & innovation, technology transfer and dissemination of results and will respond effectively to issues related to improving the energy efficiency strategy for 2020 and the longer term.

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